

QualityCore® Reference Sheet Precalculus

Triangles

Law of Sines

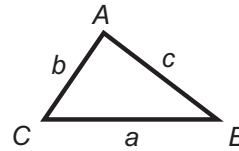
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Area of a Triangle

$$\begin{cases} \text{Area} = \frac{1}{2}bc \sin A \\ \text{Area} = \sqrt{s(s-a)(s-b)(s-c)} \end{cases}$$



$$s = \text{semi-perimeter} = \frac{(a+b+c)}{2}$$

Conic Sections

Circle

$$(x-h)^2 + (y-k)^2 = r^2$$

(h, k) = center, r = radius

Parabola,
opening vertically

$$y = a(x-h)^2 + k$$

axis of symmetry $x = h$

focus $(h, k + \frac{1}{4a})$, directrix $y = k - \frac{1}{4a}$

Parabola,
opening horizontally

$$x = a(y-k)^2 + h$$

axis of symmetry $y = k$

focus $(h + \frac{1}{4a}, k)$, directrix $x = h - \frac{1}{4a}$

Ellipse,
major axis horizontal

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1, a > b$$

foci $(h \pm c, k)$ where $c^2 = a^2 - b^2$

Ellipse,
major axis vertical

$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1, a > b$$

foci $(h, k \pm c)$ where $c^2 = a^2 - b^2$

Area of an ellipse

$$A = \pi ab$$

Hyperbola,
transverse axis horizontal

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

foci $(h \pm c, k)$ where $c^2 = a^2 + b^2$

Hyperbola,
transverse axis vertical

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

foci $(h, k \pm c)$ where $c^2 = a^2 + b^2$

Sequences and Series

Arithmetic Sequence

$$a_n = a_1 + (n-1)d$$

a_n = n th term

Arithmetic Series

$$s_n = \frac{n}{2}(a_1 + a_n)$$

a_1 = first term

Geometric Sequence

$$a_n = a_1(r^{n-1})$$

n = number of the term

Finite Geometric Series

$$s_n = \frac{a_1 - a_1 r^n}{1-r} \text{ where } r \neq 1$$

d = common difference

r = common ratio

Infinite Geometric Series

$$s = \frac{a_1}{1-r} \text{ where } |r| < 1$$

s_n = sum of the first n terms

s = sum of all the terms

Exponential Functions

Discretely
Compounded Interest

$$A = p \left(1 + \frac{r}{n}\right)^{nt}$$

A = amount of money after t years

p = starting principal r = interest rate

Continuously
Compounded Interest

$$A = pe^{rt}$$

n = compound periods per year

t = number of years $e \approx 2.718$

Discrete, Continuous
Exponential Growth

$$N_t = N_0(1+r)^t, N_t = N_0e^{rt}$$

N_t = value after t time periods

r = rate of growth t = time periods

Polar Coordinates and Vectors

De Moivre's Theorem	$[r(\cos \theta + i \sin \theta)]^n = r^n(\cos n\theta + i \sin n\theta)$	$r =$ radius, distance from origin $\theta =$ angle in standard position $n =$ exponent
Conversion: Polar to Rectangular Coordinates	$x = r \cos \theta$ $y = r \sin \theta$	
Conversion: Rectangular to Polar Coordinates	$r = \sqrt{x^2 + y^2}$, $\theta = \arctan \frac{y}{x}$, when $x > 0$ $\theta = \arctan \frac{y}{x} + \pi$, when $x < 0$	
Product of Complex Numbers in Polar Form	$r_1(\cos \theta_1 + i \sin \theta_1) \cdot r_2(\cos \theta_2 + i \sin \theta_2) =$ $r_1 r_2 [\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)]$	
Inner Product of Vectors	$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$	$\mathbf{a} = \langle a_1, a_2 \rangle$ vector in the plane $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ vector in space

Matrices

Determinant of a 2x2 Matrix	$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$	
Determinant of a 3x3 Matrix	$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & j \end{bmatrix} = a \cdot \det \begin{bmatrix} e & f \\ h & j \end{bmatrix} - b \cdot \det \begin{bmatrix} d & f \\ g & j \end{bmatrix} + c \cdot \det \begin{bmatrix} d & e \\ g & h \end{bmatrix}$	
Inverse of a 2x2 Matrix	$M^{-1} = \frac{1}{\det M} \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}$ where $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$	

Trigonometry

Sum and Difference Identities	$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$ $\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$ $\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$	$\alpha, \beta, \theta =$ angles, from positive x-axis
Double-Angle Identities	$\sin 2\theta = 2 \sin \theta \cos \theta$ $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$	
Half-Angle Identities	$\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$ $\cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$ $\tan \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}$, where $\cos \alpha \neq -1$	